



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Energy Efficient Pump Control for an Offshore Oil Processing System

*IFAC Workshop - Automatic Control in Offshore Oil and Gas Production*

Yang, Zhenyu; Soleiman, Kian ; Løhndorf, Bo

*Published in:*

2012 IFAC Workshop on Automatic Control in Offshore Oil and Gas Production Automatic Control in Offshore Oil and Gas Production

*DOI (link to publication from Publisher):*

[10.3182/20120531-2-NO-4020.00050](https://doi.org/10.3182/20120531-2-NO-4020.00050)

*Publication date:*

2012

*Document Version*

Accepted author manuscript, peer reviewed version

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Yang, Z., Soleiman, K., & Løhndorf, B. (2012). Energy Efficient Pump Control for an Offshore Oil Processing System: IFAC Workshop - Automatic Control in Offshore Oil and Gas Production. In *2012 IFAC Workshop on Automatic Control in Offshore Oil and Gas Production Automatic Control in Offshore Oil and Gas Production* (1 ed., Vol. 1, pp. 257-262). Elsevier. Elsevier IFAC Publications / IFAC Proceedings series  
<https://doi.org/10.3182/20120531-2-NO-4020.00050>

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

## Energy Efficient Pump Control for an Offshore Oil Processing System

Zhenyu Yang\* Kian Soleiman\* Bo Løhndorf\*\*

\* *Department of Energy Technology, Aalborg University, Esbjerg  
Campus, Niels Bohrs Vej 8, DK-6700 Esbjerg, Denmark (e-mail:  
yang@et.aau.dk).*

\*\* *Ramboll Oil and Gas A/S, Willemoesgade 2, DK-6700 Esbjerg,  
Denmark.*

---

### Abstract:

The energy efficient control of a pump system for an offshore oil processing system is investigated. The seawater is lifted up by a pump system which consists of three identical centrifugal pumps in parallel, and the lifted seawater is used to cool down the crude oil flowing out of a three-phase separator on one of the Danish north-sea platform. A hierarchical pump-speed control strategy is developed for the considered system by minimizing the pump power consumption subject to keeping a satisfactory system performance. The proposed control strategy consists of online estimation of some system operating parameters, optimization of pump configurations, and a real-time feedback control. Comparing with the current control strategy at the considered system, where the pump system is on/off controlled, and the seawater flows are controlled by a number of control valves, the proposed control strategy has showed significant energy savings without sacrificing the system performance.

*Keywords:* Pump control, optimization, modeling, process control, energy saving

---

### 1. INTRODUCTION

Pump systems have been extensively used in offshore oil & gas industries, for instance, in order to get the crude oil/gas out of reservoirs, transport the produced oil/gas to the onshore or nearby offshore processing platforms, and move the products from one processing facility to the next one (Maersk (2004)). There is no doubt that pump systems consume significantly large amount of energy every year (Maersk (2004); Reeves (2001); Rishel (2003)). In the report (Reeves (2001)), European Commission pointed out that the largest energy saving of pump systems can be made through the better design and control of pump systems. In order to have a good pump design for specific applications, the pump manufactures often need to closely cooperate with customers. The pump control can be regarded as a type of *soft mechanism* to further improve the pump system's efficiency (Shiels (2001); Westerlund *et al* (1994)).

For a large pump system, which often consists of a number of pumps with different capabilities, physical sizes and configuration etc, an efficient pump control system needs to cope with two type of tasks: (i) pump scheduling; and (ii) real-time pump control. The pump scheduling needs to find out the best pump configuration w.r.t. the current expectation and operating conditions, e.g., to decide how many and which pumps need to be put into or pull out from operation. The real-time control needs to guarantee a satisfactory system performance subject to different

operating conditions and disturbances (Pettersson and Westerlund (1996); Rishel (2003); Yang and Børsting (2010b); Yu *et al* (1994)).

The optimization of pump system control has been extensively studied in recent decades. A large amount of algorithms/methods have been proposed for different systems and applications. For instance, Westerlund *et al* (1994) studied the configuration optimization of multiple-pump systems by using a Mixed Integer NonLinear Programming (MINLP) method. Pettersson and Westerlund (1996) further embraced a binary separable programming method into the previous work in order to cope with the non-convex problem. However, these methods are not oriented for real-time dynamic pump scheduling. Yu *et al* (1994) proposed an optimal pump scheduling algorithm for water distribution systems, where a number of water reservoirs and pump stations are considered and modeled as nodes in a networked system model. However, the configuration of pumps within the group/station and the speed control of each running pump are neglected. Pedersen and Yang (2008); Savic *et al* (1997) employed Evolutionary Algorithms (EAs) to handle the pump optimization problem. The EA methods can cope with non-convex optimization problem. However, the development of these methods can be time-consuming w.r.t. the fact that it often requires extensive experiments and data, and the computation load can not be ignored if concerned for real-time implementation.

Recently, Yang and Børsting (2010,b) proposed a hybrid control strategy to handle a multi-pump system equipped with multiple variable-speed pumps in parallel. This hy-

---

\* Kian Soleiman's current address is Ramboll Oil and Gas A/S, Willemoesgade 2, DK-6700 Esbjerg, Denmark.

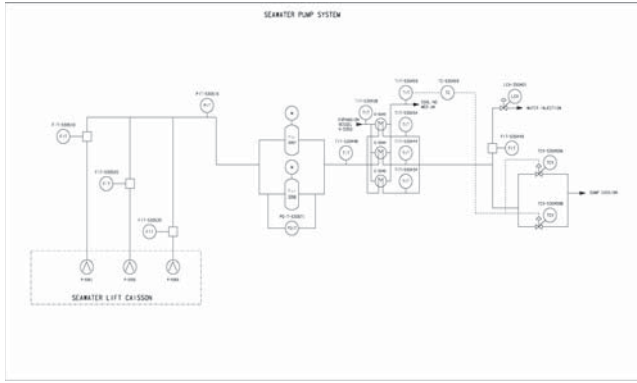


Fig. 1. Schematic diagram of the considered system

brid control solution consists of an estimation block, which is used to estimate system operating parameters in an online manner, an optimization block, which determines the best pump configuration (in terms of energy efficiency) and recommend the optimal running speeds of the selected pumps, and a feedback control block, which consists of a feed-forward control and a feedback control. The testing results from a lab-sized pump system showed that there is a huge potential to improve the pump system's efficiency without sacrificing the system performance. Thereby, here we will investigate the extension of this proposed method into a practical application, i.e., development of an energy efficient pump control for an offshore oil processing system, where a pump system with three identical centrifugal pumps arranged in parallel is used to drain the cold seawater so as to cool down the crude oil flowing out of a first-stage three-phase separator (Maersk (2004)).

The rest of paper is organized as: Section 2 presents the considered system and problem; Section 3 discusses the modeling of considered pump system and the relevant cooling system; Section 4 proposes a model-based control strategy and methods for energy efficient pump control; Section 5 illustrates some preliminary results; and finally we conclude the paper in Section 6.

## 2. CONSIDERED SYSTEM AND PROBLEM

The considered system, as shown in Fig.1, consists of a pump system, a filter system, a heat exchange system and the outlet flow control system. The pump system is employed to drain the pre-treated seawater from a seawater caisson, where the water is treated with electrolysis in order to limit the fouling problem on the pump blade surfaces. This pump system consists of three identical centrifugal pumps arranged in parallel. Each pump has a design flow capacity of  $1500\text{m}^3/\text{h}$  and the driving motor rates to  $710\text{kW}$  at this designed capacity. The whole seawater or part of that passes through a filter system, where two identical filter vessels are arranged in parallel. If necessary, the rest of drained seawater can be directly dumped back to seawater caisson. The filtered seawater enters a heat exchange system, where three identical plate-type heat exchangers are arranged in parallel. The cold seawater is used to cool down a type of cooling medium flowing through another set of closed pipelines inside the heat exchange system. The cooling medium liquid is 70% water with 30% Mono Ethylene Glycol. The cooled cooling

medium is then used to cool down the crude oil flowing out of a first-stage three-phase separator on one offshore platform. As a standard process, the mixture of crude oil and gas transported from the surrounding drilling wells/platforms often needs to be heated up before it enters the first-stage three-phase separator in order to make the separation more efficient (Maersk (2004)). A part of the seawater flowing out of the heat exchange will be used for other facilities, such as water injection etc, and the rest of water will be dumped back to seawater caisson. This is managed by control valves at the outlet flow control system.

From the control point of view, the considered system is controlled in two perspectives: (i) On/Off control of the pump system; (ii) the seawater flow control. All pumps are controlled by *on/off* commands from the SCADA system. When the command is at "on", the corresponding pump will run at its full speed. When the command is at "off", the corresponding pump will be on standby. If the pressure in the seawater system drops to  $6.5\text{ barg}$ , or if a duty pump fails, the third pump will be automatically started. The seawater flow through the entire system is controlled by a number of control valves. There are two temperature control valves (denoted as TCV in Fig.1) arranged in parallel in the outlet flow system. These linear globe type valves are controlled by a temperature controller (denoted as TC in Fig.1). This temperature controller manipulates the opening degree of these temperature control valves according to the cooling medium's temperature measurement downstream the cooling medium coolers (denoted as TIT in Fig.1). A level control valve in the downstream water injection system controls the flow to this system, based on the demand for water injection. In addition to these two major consumers, a number of minor utility and service systems are utilizing the lifted seawater, but these are negligible and not considered here.

The pump control and the temperature and level controls are developed independently from each other, and this may lead to energy waste. Ideally, in steady-state operation the amount of lifted seawater and the following flow through the cooling medium coolers should be equal to the demand for seawater to the water injection system.

A piece of recorded data from the current system regarding the seawater flows generated by the pumps and used for heat exchange are illustrated in Fig.2. For this 3-month winter period, the average seawater flow generated by two pumps is  $2078\text{ m}^3/\text{h}$ . There is  $1224\text{ m}^3/\text{h}$  of the total routed to utilities, and the rest,  $854\text{ m}^3/\text{h}$  is directly dumped overboard the platform. The waste of seawater is about 41.1% in average.

A piece of recorded data of year-around seawater temperature and the openness degree of these control valves (TCV) are illustrated in Fig.3. It can be observed that the yearly average openness of the control valves is only about 18.5%. This also indicates that a large amount of seawater lifted up by the seawater pumps is simply dumped directly.

The amount of directly dumped seawater indicates an amount of energy waste, because this amount of dumped seawater is also lifted up by the seawater pumps. In order to reduce this kind of energy waste and thereby improve the entire system's efficiency, one effective way is to use





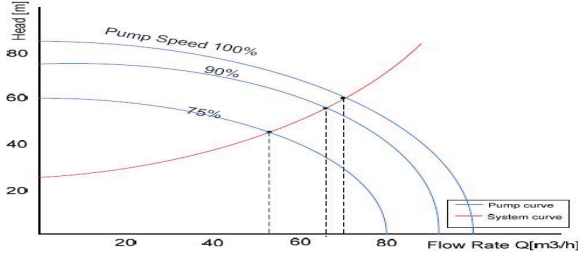


Fig. 5. Determination of system operating point according to pump curve and system curve

degrees in a water circulation system. If the control valve position is fixed, then the coefficient  $k_1$  can be simplified as constant.

### 3.2 Heat exchange Modeling

Within the three parallel identical heat exchangers, there are a number of temperature sensors used to measure the seawater and cooling medium temperatures at inlets and outlets of heat exchangers, respectively. According to the design specification, the cooling medium inlet temperature needs to be kept below  $50^\circ\text{C}$  with 8.3 barg pressure and the outlet temperature below  $27^\circ\text{C}$  with 7.3 barg pressure. According to the yearly round North Sea condition, the seawater inlet temperature is normally not over  $13^\circ\text{C}$  with 5.3 barg pressure, and outlet seawater temperature is normally not over  $32^\circ\text{C}$  with 4.3 barg pressure.

Without loss of generality, the dynamic of the heat exchanger can be modeled according to the thermal dynamic theory as

$$\begin{aligned} C_{sw}\dot{T}_{sw}(t) &= -c_{sw}\rho_{sw}Q_{sw}(t - \tau_{sw})(T_{sw_{in}}(t - \tau_{sw}) - T_{sw}(t)) \\ &\quad - \frac{1}{R}(T_{sw}(t) - T_{cm}(t)), \\ C_{cm}\dot{T}_{cm}(t) &= c_{cm}\rho_{cm}Q_{cm}(t - \tau_{cm})(T_{cm_{in}}(t - \tau_{cm}) - T_{cm}(t)) \\ &\quad + \frac{1}{R}(T_{sw}(t) - T_{cm}(t)), \end{aligned} \quad (9)$$

where the system variables and coefficients are listed in Table 1.

Table 1. System variables and coefficients of modeling heat exchanger

Symbol	Interpretation (SW: seawater) (CM: cooling medium)	value	unit
$C_{sw}$	SW's thermal capacity	$4.5732 \times 10^6$	$\frac{J}{\text{degree}}$
$C_{cm}$	CM's thermal capacity	$5.859 \times 10^6$	$\frac{J}{\text{degree}}$
$T_{sw}$	SW outlet temp.	variable	degree
$T_{cm}$	CM outlet temp.	variable	degree
$T_{sw_{in}}$	SW inlet temp.	variable	degree
$T_{cm_{in}}$	CM inlet temp.	variable	degree
$\tau_{sw}$	SW time-delay coeff	$Q_{sw}$ -depend	$h$
$\tau_{cm}$	CM time-delay coeff	$Q_{cm}$ -depend	$h$
$c_{sw}$	SW's specific heat	3.906	$\frac{J}{\text{kgdegree}}$
$c_{cm}$	CM's specific heat	3.811	$\frac{J}{\text{kgdegree}}$
$Q_{sw}$	SW flow rate	variable	$\text{m}^3/\text{h}$
$Q_{cm}$	CM flow rate	variable	$\text{m}^3/\text{h}$
$R$	thermal resistance	$4.5732 \times 10^{-7}$	$\frac{\text{degree}}{W}$

By taking  $Q_{sw}(t)$  as the control variable and  $T_{sw_{in}}, T_{cm_{in}}, Q_{cm}$  as external inputs, from (9) it is clear that the considered system is a type of nonlinear system and the

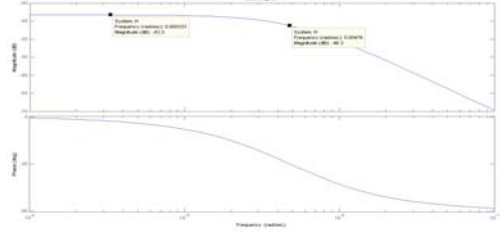


Fig. 6. Bode plot of the heat exchanger model without consideration of time-delay

nonlinearities exhibit in two perspectives: (1) there is the term of  $Q_{sw}T_{sw}/Q_{cm}T_{cm}$ ; (2) the system time delays  $\tau_{sw}/\tau_{cm}$  are flow dependent.

Under the assumption that the system runs into a steady-state operation, and all external inputs are constants, then a linearized model can be obtained at this steady-state point, and its transfer function description can be derived as

$$T_{cm}(s) = \frac{\beta_c e^{-\tau_{sw} s}}{\alpha_c(s)\alpha_s(s) - 1} Q_{sw}(s) - \frac{\alpha_c(s)\beta_s e^{-\tau_{cm} s}}{\alpha_c(s)\alpha_s(s) - 1} T_{cm_{in}}(s) - \frac{\gamma_c e^{-\tau_{sw} s}}{\alpha_c(s)\alpha_s(s) - 1} T_{sw_{in}}(s), \quad (10)$$

with the system parameters as

$$\begin{aligned} \alpha_c(s) &= RC_{sw}s - Rc_{sw}\rho_{sw}Q_{sw_0} + 1, \\ \alpha_s(s) &= RC_{cm}s - Rc_{cm}\rho_{cm}Q_{cm_0} + 1, \\ \beta_c &= -Rc_{sw}\rho_{sw}(T_{sw_{in_0}} - T_{sw_0}), \\ \beta_s &= -Rc_{cm}\rho_{cm}Q_{cm_0}, \\ \gamma_c &= Rc_{sw}\rho_{sw}Q_{sw_0} \end{aligned} \quad (11)$$

where all sub-subscript 0 indicate the corresponding variables' values at this equilibrium point. It is clear that (10) is a type of second-order system with proper dead-times, so that some standard control design method can be potentially employed to develop a temperature controller. The bode plot of this linearized system (without the time-delay) is shown in Fig.6, it is quite clear that this dynamic system is very slow with a bandwidth of 0.00164Hz.

### 3.3 Entire System Model

The entire system model can be achieved by combining the static pump system model (5) with the dynamic heat exchanger model (9), as illustrated in Fig.7. all the external inputs to the heat exchanger model are practically measured, thereby they are regarded as known external inputs. The pump system model need to be determined according to the pump configuration (e.g., number of pumps and their structure and speed configurations), and the system curve coefficients  $k_0, k_1$ , which can be identified in an off-line manner (Yang and Børsting (2010)).

## 4. CONTROL STRATEGY

Similar as the strategy we proposed for energy efficient control of a group of parallel pumps in Yang and Børsting (2010,b), hereby a hierarchical control structure is proposed as shown in Fig.7. The high-level controller consists of an estimation and optimization block dedicated for the efficient pump control, and this block only need

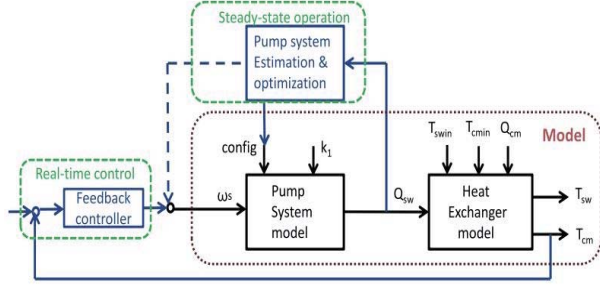


Fig. 7. Diagram of considered control structure and the system model

to be operated when the entire system runs into steady state operation. The low-level controller consists of a real-time feedback controller, which controls the speeds of the selected pump configuration so as to keep the cooling medium temperature at the expected set-point. If necessary, the high-level controller could also provide some reference pump speeds, then this block will also act as a kind of feed-forward controller, besides the determination of the best pump configuration in terms of energy efficiency.

#### 4.1 Pump Estimation and Optimization Block (PEOB)

This PEOB dedicates to two objectives: (i) Estimate the system curve coefficients  $k_0$ ,  $k_1$  in an (steady-state) online manner; (ii) Determine the best pump configuration under the current steady-state operating condition in terms of minimizing the pump energy consumption, subject to maintaining the required system performance. It should be mentioned that, due to the fact that the static pump model (5) will be used to derive solutions for both objectives, the PEOB only makes sense when the considered system runs into steady state operation.

**Pump Curve Estimation** The estimation of system curves can be achieved by following simplified steps under assumption that there are identical pumps in parallel (Yang and Børsting (2010b)):

- *Initialization*: the current pump combination, the pumps' operating speed and the steady state (total) flow (denoted as  $Q_1$ ) need to be given beforehand or they are measurable;
- *Step 1*: according to the pump model (5) corresponding to the current configuration, determine the operating pump head (denoted as  $H_1$ ) by using the known pump speed. The pair of  $(H_1, Q_1)$  is one sample to be used for system curve estimation;
- *Step 2*: Manage to operate the considered system to a slightly different steady state operating point by some deliberative way, for example, to switch off or on the feedback control, or to change the pump speed manually, and then wait until the system runs into another steady state point. Record the pump flow (denoted as  $Q_2$ ) under this steady state point and also calculate the corresponding pump head (denoted as  $H_2$ ). The pair of  $(H_2, Q_2)$  is another sample to be used for system curve estimation;
- *Step 3*: Both  $k_0$  and  $k_1$  can be estimated from

$$k_1 = \frac{H_1 - H_2}{Q_1^2 - Q_2^2}, \quad k_0 = H_1 - k_1 Q_1^2. \quad (12)$$

**Configuration Optimization** The solution for the best pump configuration needs to solve a constraint MINLP problem (Yang and Børsting (2010b)), which is defined as :

$$\min_{\substack{N \in \{1, 2, \dots, N_{max}\} \\ \omega \in \{0, \omega_{max}\}}} P_s(\omega), \quad (13)$$

subject to the constraint

$$H_{s0} = a_0^s \omega^2 + a_1^s \omega Q_{s0} + a_2^s Q_{s0}^2, \quad (14)$$

where coefficients  $a_0^s, a_1^s, a_2^s$  are from (2),  $H_{s0}$ ,  $Q_{s0}$  are the pump system head and flow at the current steady state point. If  $H_{s0}$  is not directly measured, it can be estimated by using the obtained system curve (5) and the measured flow.

Thanks to the identical pump assumption, the optimization problem (13) can be easily solved by enumerating all different pump configurations and predicting the corresponding energy consumption of each configuration using (5), then the configuration which leads to least energy consumption is the best solution. The solution for a general case can be found in Yang and Børsting (2010b).

#### 4.2 Real-Time Feedback Controller

The objective of the feedback controller is to maintain the cooling medium temperature close to the expected set-point, subject to some potential modeling errors and unknown disturbances. This controller can be designed according to some standard feedback control design methods, such as PID control and tuning. Nevertheless, this design needs to take care of the system time-delay and nonlinearities in the considered system. If the feed-forward control structure is intended to be used, where the feed-forward speed signal is the speed solution ( $\omega$ ) of problem (13), the design of feedback controller can be simply only based on the heat exchanger model (9) or the linearized model (10).

## 5. SOME PRELIMINARY RESULTS

A number of PID controllers are developed for the cooling medium temperature control. One set is obtained based the standard Ziegler-Nichols method, and the simulation results showed that this controller can increase the closed-loop system bandwidth by 10 times (up to 0.0108Hz), meanwhile the overshoot is controlled within 18%. Another set of controller is developed and under testing, based on the Internal Model Control (IMC) method. The basic idea is illustrated in Fig.8. Here the extension comparing with standard IMC development is that the static model of the pump system need to be cooperated into the controller (can be regarded as a type of checking table).

From the energy efficient point of view, one scenario consider the set-point of the cooling medium temperature at 20°C, under a specific (steady state) operation condition, it is concluded that to operate two pumps at a common speed is the best configuration, where the recommend speed is 64% of the full speed, with the total flow of 1916m<sup>3</sup>/h. From the power prediction as shown in Fig.9,

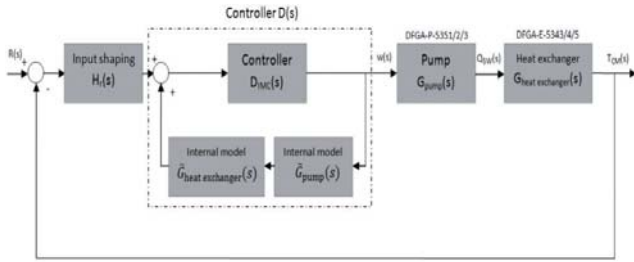


Fig. 8. Diagram of feedback control design using IMC principle

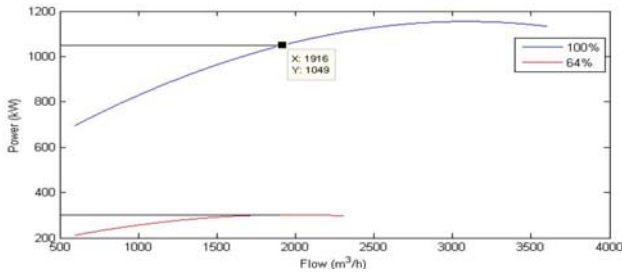


Fig. 9. Predictions of pump power consumptions using pump model (5)

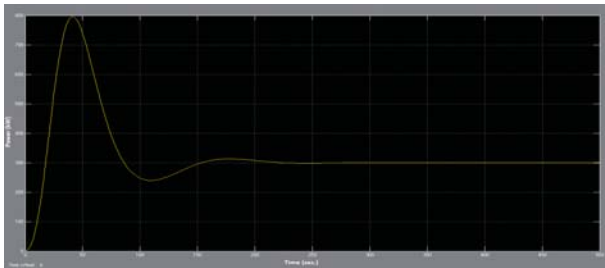


Fig. 10. Dynamic power consumption along with system operation

the pump system power consumption at that corresponding speed is  $302.5\text{ kW}$ , while the power consumption of two pumps at full speeds (which is the current real situation on platform) is up to  $1049\text{ kW}$ . The energy saving can be up to 71.2% by using variable speed drivers and feedback control, comparing with the current operation.

The real-time power consumption of the pump system (from simulation) in the considered scenario is shown in Fig.10, where power consumption varied before it settled down at  $302.5\text{ kW}$ . This variation is mainly due to the varying speed of the pumps in order to track the set-point. Nevertheless, comparing with the power consumption under the current situation (full speed, varied from  $1008\text{ kW}$  to  $1123\text{ kW}$ ), the potential power saving is still quite significant. Of course, in the simulation study, we didn't consider about the power consumption of the frequency converter and other electricity consumptions, and the heat exchange model has not yet been completely validated. Furthermore, due to the safety reason, the proposed methods have not yet been tested in the real facilities. We expect to report more latest testing results in the final camera-ready version if this work is accepted.

## 6. CONCLUSION

The current control strategy in the considered system employs (i) on/off control of pump system; (ii) control valves for seawater flow control. It has been observed that the current control strategy sometimes leads the entire system to waste a huge amount of energy, especially the electricity energy consumed by the pump system.

By recommending to use variable speed pump systems, an energy efficient control strategy is proposed for controlling the pump system in a manner of minimizing the pump power consumption subject to keeping a satisfactory system performance. The proposed control strategy consists of online estimation of pump system coefficients, optimization of pump configurations and speeds, and a real-time feedback control in order to handle some modeling errors and disturbances. Comparing with the current control strategy at the considered system, through the simulation study, the proposed control strategy has already showed significant energy savings without sacrificing the system performance. The implementation in the real setup and testings will be part of our future work.

## REFERENCES

- Maersk Oil Gas A/S. Production operational procedure. *DF-53-03*, 2004.
- G.K. Pedersen and Z. Yang. Efficiency Optimization of a Multi-pump Booster system. *Proc. of Genetic and Evolutionary Computation Conference (GECCO-2008)*, Atlanta, Georgia, USA, Jul 12-16 2008, pp.1611-1618
- F. Pettersson and T. Westerlund. Global optimization of pump configurations using binary separable programming. *Computers & Chemical Engineering*, Vol.21, No.5, 1996, pp.521-529.
- D. Reeves. Study on improving the energy efficiency of pumps. *European Commission, AEAT-6559/v5.1.*, [http://re.jrc.ec.europa.eu/energyefficiency/motorchallenge/pdf/SAVE\\_PUMPS\\_Final\\_Report\\_June\\_2003.pdf](http://re.jrc.ec.europa.eu/energyefficiency/motorchallenge/pdf/SAVE_PUMPS_Final_Report_June_2003.pdf), 2001.
- J. Rishel. Control of variable speed pumps for HVAC water systems. *ASHRAE Transactions*, Vol.109, No.1, 2003, pp. 380-389.
- D.A. Savic, G.A. Walters and M. Schwab. Multiobjective genetic algorithms for pump scheduling in water supply. *LNCS*, Vol.1305, 1997, pp.227-235.
- S. Shiels. Optimizing centrifugal pump operation. *World Pumps*, Jan 2001, pp.35-39.
- T. Westerlund, F. Pettersson and I.E. Grossmann. Optimization of pump configurations as a MINLP problem. *Computers & Chemical Engineering*, Vol.18, No.9, 1994, pp.845-858.
- Z. Yang and H. Børsting. Optimal Scheduling and Control of a Multi-Pump Boosting System. *Proc. of IEEE CCA2010*, Yokohama, Sept. 8-10, 2010, pp.2071-2076.
- Z. Yang and H. Børsting. Energy Efficient Control of a Boosting System with Multiple Variable-Speed Pumps in Parallel. *Proc. of IEEE CDC2010*, Atlanta, Georgia USA, December 15-17, 2010, pp.2198-2203.
- G. Yu, S. Powell and M.J.H. Sterling. Optimized pump scheduling in water distribution systems. *Journal of optimization theory and applications*, Vol.83, No.3, Dec 1994, pp. 463-488.